

Asset Management by Derivation

Remaining life, maintenance decisions and the value of condition monitoring

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Asset Management by Derivation

Volume II

From the asset manager's questions to probabilistic inference and decisions

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How to read this book

Volume I of this book holds Parts I to III, Chapters 1 to 13: the method itself (a plant graph, a question as a probability over its state, and the five computations beneath every answer: the posterior, the trend, the evidence, the preposterior and the expected consequence), the condition of the hall's coils, its battery string and a fleet of fifty coils today, and the rates at which the coils' condition changes, read from three years of monthly windows. This second volume holds Parts IV to VI, Chapters 14 to 24. It turns from how fast to when and what: Part IV forecasts the need for attention (the date the coils can no longer hold the hall, the probability of a failure in a mission window, and a fleet's failure rate by age); Part V makes the maintenance decisions as expected consequences over that one posterior (repair or replace, the inspection interval, the trigger for a work order, the order in which one crew visits fifty units, and a warranty claim); and Part VI values the monitoring itself before it is logged (whether the sensors can resolve a stated loss in the time available, which sensor narrows the crossing date most, and how often to log). Chapter numbers run continuously across the two volumes; a reference to a chapter numbered 13 or below points to Volume I, whose results this volume builds on and does not repeat.

Every chapter follows the progression fixed in Volume I's preface: step 0, the plant graph; step 1, the question as a probability or expectation over the state; step 2, the graph extended for the question and the evidence attached as rows; step 3, the posterior of the state, derived and computed once; step 4, the answers, each read from that posterior down to its number; step 5, the checks against the full calculation and against the record's constructed truth; step 6, the report with its band and the entry in the maintenance log; step 7, the next measurement, ranked by the width it removes. The notation chapter below is the one of Volume I with this volume's symbols added, and the atlas of questions in the appendix collects the rows of both volumes.

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Notation and vocabulary

Every symbol below is used with the same meaning in every chapter. The unknown always stands on the left of a conditioning bar and the evidence on the right. Vectors and matrices are bold. Coil subscripts are $u \in \{1, 2\}$, rack subscripts $j \in \{1, 2, 3\}$ and fleet subscripts $i = 0, \ldots, 49$; windows are months, indexed t.

Symbol	Unit	Meaning
The plant graph and its state		
$\mathbf{z}$, z_i	varies	State vector of the plant graph: every free quantity estimated at once; its i-th entry.
$\mathbf{z}_t$, t	varies, –	The state of window t; the window index, $t = 0$ the commissioning test and $t = 1, \ldots, 36$ the months from January of year 1 to December of year 3.
$\mathbf{y}$, y	varies	The evidence: all readings, test results, counts and prior information treated as known; a single reading, subscripted by what it reads.
$r_k(\mathbf{z})$	varies	Row k of the model: a physical relation that must hold ($r_k = 0$), a reading compared with the state, a trend row, or a prior. As a subscript k is this row (and in Chapter 24 a channel's rows); in Chapters 2, 3 and 15, whose bare k is the count of failures, the row is ℓ.
w_k, σ_k	varies	Weight (precision) of row k and the standard deviation it corresponds to, $w_k = 1/\sigma_k^2$.
σ_p	varies	Tolerance of a physics row; an exact constraint is the limit $\sigma_p \to 0$.
The equipment hall of Part I		
T, T_c	°C	Temperature of the hall air; the supply air held by the cooling unit's discharge controller, pinned.
Q, G	kW, kW/K	Heat load of the hall; effective conductance of its air path to the supply air.
g, G_{ref}	–, kW/K	Log-conductance $\ln(G/G_{\text{ref}})$, the entry of the state that carries the conductance; the design conductance on the design sheet, 6 kW/K.
h, h^*, h_F	–	Part I's health index $h = G/G_{\text{ref}} = \mathrm{e}^g$, the conductance against design; the index a requirement needs; the functional limit, at which the asset stops doing its job. From Part II the health index is against commissioning, $h_{u,t}$ below.
μ_g, σ_g	–	Mean and spread of the handbook prior on g, $\mu_g = \ln(7.5/6)$.

Symbol	Unit	Meaning
g_t, $\hat{g}_0$	–	The log-conductance in window t; the commissioned value, the fit of the commissioning test (window 0).
$\sigma_{\hat{g}_0}$, $\hat{G}_0$	–, kW/K	Spread of the commissioning row on g (the commissioning fit carried forward); the commissioned conductance $G_{\text{ref}}\text{e}^{\hat{g}_0}$.
G_{test}, h_{test}	kW/K, –	The conductance and index one condition test reports when its reading is taken as exact, $Q/(y_T - T_c)$.
g^*, g_F	–	A requirement level on g, $\ln h^*$; from Chapter 19, the level on each coil's log-derate at which two equal coils give G^*, $\ln(G^*/(\bar{G}_1 + \bar{G}_2)) = -0.26313$; the functional limit, $\ln h_F$, in Chapter 20 $\ln(5/6) = -0.18232$ on each fleet coil's log-derate against rating.
$T_{\max}$, T^*	°C	A temperature limit: 24°C now and 27.5°C at 60 kW (the owner's request, Chapter 1), 28°C (the commissioning record's acceptance limit, Chapter 4); a threshold on a test temperature.
Q_2, T_2	kW, °C	The requested load and the hall temperature it produces, a scenario node.
ρ, μ_0, σ_0	K/kW	Thermal resistance of the air path, $1/G$; the commissioning record's pre-test estimate of it and its spread (Chapter 4).
b_T	K	Calibration offset of the hall sensor, a nuisance quantity or a bias entry.
Δ_{m}, σ_{m}	kW	A model discrepancy of the physics row and its spread (Chapter 1).
C_{th}, ϑ, ϑ_{th}, ΔT	kJ/K, s or h, s, K	Thermal capacitance of the hall; elapsed time within a record (seconds in Chapter 1's transient, hours into a battery test in Chapter 8, hours within a window in Chapter 24); the hall's time constant C_{th}/G; the departure from steady state.
y_T, σ_T	°C, K	The hall sensor's reading and the standard deviation of its error.
The two-unit hall of Parts II–VI		
$T_s(t)$, $Q_o(t)$	°C, kW	Chilled-water supply temperature and the hall's other heat in window t: known inputs that follow the season.
Q	kW	IT heat load of the hall, an entry of the state read by the hall load meter.
G_u, $\bar{G}_u$	kW/K	Effective conductance of coil u; its rated value, 7 kW/K.
g_u, d_u	–	Log-derate $\ln(G_u/\bar{G}_u)$, the entry that carries coil u's conductance against its rating; the derate factor $d_u = \text{e}^{g_u}$, one at the rating.
$g_{u,t}$, $d_{u,t}$, Q_t	–, –, kW	The log-derate, derate factor and IT load in window t.
$h_{u,t}$	–	Health index of coil u in window t against its commissioned value, $G_{u,t}/G_{u,0} = \text{e}^{g_{u,t}-g_{u,0}}$ (Part II on).
$A_{u,t}$, h_u^{raw}	K, –	Approach of unit u in window t, $T_{d,u} - T_s(t)$; the dashboard's raw-performance index $A_{u,0}/A_{u,t}$, the naive baseline.
$G_{u,\text{test}}$	kW/K	Conductance of coil u with the window's readings taken as exact.

Symbol	Unit	Meaning
σ_W	kW	Standard deviation of a water-side heat meter's reading on a coil's duty.
$g_{u,0}$	–	The commissioned value of g_u, the fit of window 0.
C_u	kW/K	Air capacity rate of unit u, pinned at its fixed fan speed; read by an airflow station only in Chapter 23's candidates.
T_r, T_c, $T_{d,u}$	°C	Return air, cold aisle (the supply air of Part I's hall, now made by two coils), and discharge air of unit u.
T_j, β_j	°C, –	Inlet temperature of rack j; its recirculation fraction.
G^*, f^*	varies	A requirement level: G^* the conductance a duty needs (in Part I, 5.0 kW/K for 24°C at 30 kW and 6.316 kW/K for 27.5°C at 60 kW; from Part II, the total $G_1 + G_2$ of the capacity scenario); f^* a level on any derived quantity. The requirement holds on the safe side of f^*: $f \geq f^*$ for a conductance, $f \leq f^*$ for a temperature.
b_j	varies	Bias of reading j, an entry freed when a sensor is suspected of drifting.
$\check{X}$	as X	The value of X at the scenario node of the commissioned model: today's load and inputs with the commissioned coils ($\check{T}_r$, $\check{D}_u$, $\check{A}_u$; Chapter 6).
$D_{u,t}$	kW	Duty of unit u in window t, the heat its coil passes to the chilled water, $G_{u,t}(T_{r,t} - T_s(t))$.
$\eta_{A,u}$, $\eta_{D,u}$, η_s	K, kW, K	Excess approach, duty shift and supply penalty against the commissioned model at today's conditions (Chapter 6).
$A_{r,t}$, T_c^{sp}	K, °C	Return approach $T_{r,t} - T_s(t)$; a cold-aisle set point.
$G_{\mathrm{tot},t}$, G_t^{app}, $\bar{Q}_o$	kW/K, kW/K, kW	The hall's total conductance $G_{1,t} + G_{2,t}$; its apparent conductance, the metered IT load over the return approach, $y_{Q,t}/(y_{T_r,t} - T_s(t))$, a raw figure; the mean other heat, 8.00 kW (Chapter 13).
ζ_0, s_{season}, s_{load}	K/kW, K, K	The cold aisle's rise per kW at the commissioned coils; the season's and the load's terms of the cold aisle's change since commissioning; from Chapter 13, the season's and the load's terms of a raw figure's change between any two windows.
$b_{d,1}$, σ_b, $\sigma_{\ln h}$	K, K, –	Drift of unit 1's discharge sensor since commissioning (a bias entry); the prior spreads of a freed bias and of a freed change $\ln h_{u,t}$ (Chapter 7). From Chapter 10 the drift is a function of the window, $b_{d,1}(t)$.
$\dot{b}_{d,1}$, t_b, $\sigma_{\dot{b}}$	K/yr, window, K/yr	Drift rate of unit 1's discharge sensor, one entry for the record, with $b_{d,1}(t) = \dot{b}_{d,1}\,(t - t_b + 1)\,\Delta t$ for $t \geq t_b$ and zero before; the drift's first window, $t_b = 20$; the drift rate's prior spread, 2.0 (Chapter 10).
M_{neither}, M_{foul}, M_{drift}, M_{both}	–	The four drawings of Chapter 7: neither term, the derate term, the bias term, both.
K_{kk}, r_k^{N}, n_r	–	Leverage of row k, the diagonal of the hat matrix $\mathbf{K} = \mathbf{J}\boldsymbol{\Sigma}\mathbf{J}^{\mathsf{T}}$, with redundancy number $1 - K_{kk}$; the normalized residual $r_k/\sqrt{1 - K_{kk}}$; the number of rows.

Symbol	Unit	Meaning
ψ_k, $s_{d,1}$, y_b	–, K, K	The ratio in the closed-form Bayes factor of a bias on reading k (Chapter 12's ψ extends it to a step's column); the shift of unit 1's discharge reading the two suspects make; a side-by-side check's reading of a bias.
Time, trend and dates		
Δt	yr	The spacing of two windows, one month.
γ, γ_u, $\dot{\gamma}_u$	yr^{-1}, yr^{-1}, yr^{-2}	Trend (degradation rate) of a parameter; coil u's rate, the trend entry of its log-derate path, read as $100(\mathrm{e}^{\gamma_u}-1)\,\%/\mathrm{yr}$ (Chapter 10), and in a drawing that frees the curvature the rate at the record's center $\bar{t}_\mathrm{y}$; its curvature (acceleration), the change of the rate per year (Chapter 11). Chapter 9's γ_i indexes the fleet's coils instead: its γ_1 is fleet coil 1, not the hall's coil 1.
t_y, $\bar{t}_\mathrm{y}$, $\gamma_u(t_\mathrm{y})$	yr, yr, yr^{-1}	Time since commissioning, $t_\mathrm{y} = t\,\Delta t$ (Chapter 11); the record's center, 1.5 yr (window 18); coil u's rate at time t_y, $\gamma_u + \dot{\gamma}_u(t_\mathrm{y} - \bar{t}_\mathrm{y})$, so that $\gamma_u(3)$ is the rate today.
x_t, $\dot{g}_{u,t}$, $\sigma_{\dot{\gamma}}$	yr, yr^{-1}, yr^{-2}	The curvature's lever on the increment from window t to $t+1$, $(t+\frac{1}{2})\Delta t - \bar{t}_\mathrm{y}$; the path's monthly rate $(g_{u,t+1} - g_{u,t})/\Delta t$; the curvature's prior spread, 0.05 (Chapter 11).
t_knee	window	The knee of a rate, after which it changes by $\dot{\gamma}_u$ per year: 12 or 24 in Chapter 11's sensitivity forms.
Γ	yr^{-1}	The hall's conductance rate, $\ln(G_{\mathrm{tot},36}/G_{\mathrm{tot},0})/(36\,\Delta t)$ (Chapter 13). With an argument, $\Gamma(\cdot)$ is the gamma function (Chapter 2).
α	–	Level entry of a straight-line trend at the record's center $\bar{t}$.
σ_q	–	Spread of a trend row: one monthly window's departure of a parameter from its straight line, $\sigma_q\sqrt{n_\mathrm{w}}$ across n_w windows without a reading; the straight line is the limit $\sigma_q \to 0$.
$r_{q,u,t}$, $\mathbf{D}_t$, $\mathbf{E}$, $\mathbf{F}$	–, varies	Coil u's trend row from window t to $t+1$; the trend rows' blocks of the stacked Jacobian in window t's columns and in the trend entries (γ_1, γ_2) (Chapter 10), and in the curvatures (Chapter 11).
n_w, n_z, t_as	–, –, window	The number of monthly increments a rate is read over, 36 for the whole record, or of unread windows a trend row spans (Chapters 4 and 10); the number of entries of the state (Chapters 3, 4, 7 and 12); the last window of an as-of fit, the record cut at window t_as.
s_u^raw, $\dot{s}_u^\mathrm{raw}$	yr^{-1}, yr^{-2}	Slope of the dashboard's least-squares line through the 37 tiles $h_{u,t}^\mathrm{raw}$ (Chapter 10); the dashboard's bend, twice the quadratic coefficient of the least-squares quadratic through them (Chapter 11).
$\tilde{\mathbf{z}}^0$, $\tilde{\mathbf{z}}^g$	varies	The stacked state with every window's coils at the commissioned values and no drift; with the fitted coils and no drift: the steps of Chapter 10's sequential split of the dashboard's slope.

Symbol	Unit	Meaning
γ_u°	yr^{-1}	Pseudo-true value of γ_u: the drawing's fit to the truth's noise-free readings (step 5, Chapter 10).
M_0, M_1, M_2, M_{12}	–	The constant-rate drawing (Chapter 10's base fit) and the drawings that free coil 1's, coil 2's and both curvatures (Chapter 11). Chapter 12 writes the first M_{none}.
δ, δ_j, t_0	–; –, K or kW; window	Size of a step change in a parameter (with an argument, $\delta(\cdot)$ is the Dirac delta, Chapter 3); a step of kind j (Chapter 12): δ_1, δ_2 a coil's, δ_{12} both coils', δ_{T_r}, $\delta_{T_{d,1}}$, $\delta_{T_{d,2}}$, δ_{T_1} a temperature sensor's, δ_Q the load meter's; the window at which it occurs.
s, σ_δ, Π_s	window, as δ_j, –	A candidate window of a step, 1 to 36; the step's prior spread, 0.15 on a coil, 1.0 K on a temperature sensor, 10 kW on the meter; the location prior, the prior probability of a step's window s (Chapter 12).
M_{none}, $M_{j,s}$, $\mathbf{a}_{j,s}$	–, –, varies	The drawing with no step (Chapter 10's base fit); the drawing with one step of kind j into window s; the step's column in the normalized rows, $-1/\sigma_q$ on a coil's trend row into s and $1/\sigma_k$ on a sensor's readings from s on (Chapter 12).
$B_{j,s}$, $\bar{B}_s$, $\bar{B}$, $B_{\max}$	–	Bayes factor of $M_{j,s}$ against M_{none}; its mean over the eight kinds at window s; its mean over all 288 candidates; the best candidate's (Chapter 12).
ψ, ψ_a	–	A step's prior variance over its variance from the other rows, $\sigma_\delta^2\,\mathbf{a}^\mathsf{T}(\mathbf{I}-\mathbf{K})\mathbf{a}$; the same after a candidate measurement a, $\sigma_\delta^2/\sigma_{\text{dat},a}^2$ (Chapter 12).
D_h, D_γ, $\mathbf{g}_{D_\gamma}$, $\mathbf{g}_{\text{split}}$	–, yr^{-1}, –, mixed	The difference $\ln h_{2,18} - \ln h_{1,18}$ of the two coils' indices (Chapter 5); the difference $\gamma_1 - \gamma_2$ of the two rates; its gradient; the gradient of coil 2's two parts of loss, $36\,\Delta t\,\gamma_2 + \delta_2$ (Chapter 12).
σ_{dat}^2, $\sigma_{\text{dat},a}^2$, σ_{new}^2	–	A step's data-only variance from the present record, with the prior taken out; the same after a candidate measurement a; that of the candidate's rows alone (Chapter 12, step 7).
$\tilde{\delta}$, $\tilde{\delta}_0$, $\tilde{\delta}_{\text{new}}$, $\tilde{\mu}$, $\tilde{v}$	–	The mean of a step's likelihood, the estimate the rows alone give; the present record's; the candidate rows'; the combined estimate's mean and variance under the posterior predictive (Chapter 12, step 7).
t_1, t_2	window	The two windows of a raw figure's change; for the summer drop, July of year 3 (window 31) and today (window 36) (Chapter 13).
ϕ_t, mo_t	rad, –	The season's phase of window t, $2\pi(\mathrm{mo}_t - 0.5)/12$, with $\cos\phi_t = 1$ between January and February; the month index of window t, 0 for January. Window 0 takes zero seasonal regressors (Chapter 13).

Symbol	Unit	Meaning
Y_t, y_t, $\check{Y}_t$, $\check{Y}_t^{\mathrm{m}}$	as the figure	The model value of a raw figure ($T_{r,t}$ or $\ln G_t^{\mathrm{app}}$) at window t; its reading; its value with the coils at the commissioned ones, at the window's own load and inputs; that value at the season's mean inputs, window 0's (Chapter 13).
s_{coils}, s_{err}	K or –	The coils' term and the reading errors' term of a raw figure's change from window t_1 to t_2, beside s_{season} and s_{load}; in bold, each term's column over the windows (Chapter 13).
$\mathbf{X}$, $\mathbf{X}^+$, $\hat{\mathbf{k}}$, k_c, k_s	varies	The design of a least-squares regression on the windows' times (Chapters 2, 11 and 13); its pseudo-inverse $(\mathbf{X}^{\mathsf{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathsf{T}}$; the fitted coefficients; a seasonal regression's cosine and sine coefficients (Chapter 13).
$\mathbf{y}^{\mathrm{raw}}$, $\mathbf{w}$, $\mathbf{w}^{\mathrm{line}}$, $\mathbf{w}^{\mathrm{seas}}$	as the figure, yr^{-1}	The readings of a raw figure in windows 0 to t_{as}; the slope row of a regression, the second row of $\mathbf{X}^+$, without and with an annual harmonic (Chapter 13).
s^{reg}, s^{line}, s^{seas}, $s^{\mathrm{seas}}_{\mathrm{drop}}$	varies	The slope of a regression; of the line and of the seasonal regression through a raw figure, per year; the seasonal regression's reading of the summer drop, its fitted trend over the five months plus the residuals' change (Chapter 13).
$\hat{\nu}_t$, $\hat{\sigma}$, $\mathrm{se}(\cdot)$	as the figure	A regression's residual at window t; the residuals' standard deviation; the practitioner's standard error from the residual scatter (Chapter 13).
$\mathbf{\Sigma}_{31,36}$, $\mathbf{g}_{31,36}$	–	The covariance block of $(g_{1,31}, g_{2,31}, g_{1,36}, g_{2,36})$; the gradient of the coils' term of the summer drop on those entries (Chapter 13).
g_u^c, g_u^s, $\mathbf{g}^{\mathrm{sea}}$, σ_{sea}	–	Cosine and sine amplitudes of coil u's seasonal conductance, on top of its trend; the four amplitudes as one vector; their prior spread, 0.05 (Chapter 13).
M_{base}, M_{sea}	–	The base drawing (Chapter 10's base fit) and the seasonal-conductance drawing (Chapter 13).
t^*, t_c	window	The window at which a parameter's path crosses its requirement level: the remaining useful life is t^* less the present window. From Chapter 14 it is the first passage of $G_{\mathrm{tot},t}$ to G^* on the extended state, read in continuous windows by interpolation between $t_c - 1$ and t_c, with $t_c = \lceil t^* \rceil$ the first window at or below the limit, the date; from Chapter 17, $t_c(\mathbf{z}; \mathcal{P}_{\mathrm{v}})$ is counted from a visit.
t_{sd}	window	A planning date: the October shutdown of year 4 (window 46).
$\mathcal{C}_0$, $\mathcal{C}_1$, j, $M = (\mathcal{C}_l, j)$	–	The two factors of the forecast family (Chapter 14 on, through Chapter 23): the curvature state, $\mathcal{C}_0$ constant rates (Chapter 10's base fit) or $\mathcal{C}_1$ coil 1's curvature $\dot{\gamma}_1$ free (Chapter 11's drawing); the March step's kind at window 15, $j \in \{\mathrm{none}, 1, 2, 12, T_r, T_{d,1}, T_{d,2}, Q, T_1\}$ (Chapter 12's); a drawing of the family, one of 18.

Symbol	Unit	Meaning
$\mathbf{z}_{\text{ext}}$, $\mathbf{g}_t$, t_{hor}	varies, –, window	A drawing's state extended by the forecast windows $t = 37, \ldots, t_{\text{hor}}$, which carry trend rows and no reading; the two log-derates $(g_{1,t}, g_{2,t})$ of window t; the forecast horizon, window 276 (Chapter 14).
$e_{u,t}$, $\mathbf{J}_{\text{f}z}$, $\mathbf{J}_{\text{ff}}$	–, varies	A forecast window's departure from the trend, $\sigma_q\, r_{q,u,t}$; the forecast rows' Jacobian blocks on the record's entries and on the forecast entries, the second square and lower bidiagonal (Chapter 14).
Failures and the fleet		
λ, τ	yr^{-1}, yr	A constant failure rate; the length of a mission window (Chapters 2, 3, 15 and 16).
k, T_{tot}	–, unit-yr	A count of failures or corrective work orders; the exposure behind it.
ω, λ_{ref}	–, yr^{-1}	Log-rate $\ln(\lambda/\lambda_{\text{ref}})$, the entry that carries a rate; its reference, 0.1 per year.
σ_ω, T_f, K_τ	–, yr, –	Posterior spread of the log-rate; time to a unit's next corrective failure; count of corrective work orders in a period τ.
F, H, S	–	Events of a diagnosis: the coil fouled; the condition flag fires; an inspection grade, of value s in $\Pr(S = s)$ (Chapter 3).
F_A, F_B, q_S, q_A, q_B	–	Two findings or replacements; the probabilities of a shared cause and of each one's own wear.
a, b, a_λ, b_λ	–, yr	Shape and rate of a gamma prior on a rate (primed after the record); the context tells them from an action a. From Chapter 15, where a is the age, they are written a_λ, b_λ: the prior is a count row of a_λ failures in b_λ unit-years.
$\lambda(a)$, λ_0, θ_a	yr^{-1}, yr^{-1}, yr^{-1}	Failure rate at age a, $\lambda_0 \text{e}^{\theta_a a}$, the rate of occurrence of failures under minimal repair (Chapter 16); its value at age zero; its log-slope in age, a scalar.
n_a, E_a	–, coil-yr	Failures and exposure of the fleet in age bin a, the ages a to $a + 1$ (Chapter 16).
μ_γ, τ_γ	yr^{-1}	Mean and spread of the fleet's degradation rates, the shared prior of partial pooling. In Chapter 9 $\tau_\bullet$ stands for either of the fleet's spreads, τ_γ or τ_{c}, and $n_{\text{coil}} = 50$ counts the coils.
μ_{c}, τ_{c}	–	Mean and spread of the fleet's commissioning values.
$g_{i,0}$, γ_i, y_{ij}, a_{ij}, σ_{test}, k_i	–, yr^{-1}, –, yr, –, –	Commissioning value and fouling rate of fleet coil i; its j-th annual condition test and the age at it; the test's spread (0.04); its number of tests.
ℓ_{c}, ℓ_γ, $\varepsilon_{\text{c},i}$, $\varepsilon_{\gamma,i}$	–	$\ln \tau_{\text{c}}$, $\ln \tau_\gamma$, the entries that carry the spreads; the standardized deviations of the non-centered parametrization.
$\mathbf{\Omega}$, $\mathbf{\Lambda}_i$, $\mathbf{\Lambda}_\mu$, $\mathbf{\Lambda}_{\text{pr}}$, $\mathbf{\Sigma}_i$	varies	Population covariance $\text{diag}(\tau_{\text{c}}^2, \tau_\gamma^2)$; coil i's information given the spreads and means; the information on the means; their prior information; coil i's covariance with the means integrated (Chapter 9).

Symbol	Unit	Meaning
Ψ, ϖ_m	–	The Gaussian rows' sum of squares at the conditional minimum; the weight of node m of a grid on the state: the spreads' grid (Chapters 9 and 20), the log-rate's (Chapter 15), the age slope's (Chapter 16).
π_i, $\mathcal{K}_i$, $\tilde{\gamma}_0$	–	Fleet position: coil i's percentile in the design's population of rates and its rank among the fifty (1 the fastest); coil 0's standardized rate.
γ_{new}, $\hat{\gamma}_0^{\text{chart}}$	yr^{-1}	The rate of a new coil of the design (the posterior predictive); the fleet chart's slope of coil 0.
$p_\tau(\lambda)$, $\bar{p}_\tau$, R_{pred}	–	The failure probability over a window τ at the rate λ, $1 - \mathrm{e}^{-\lambda\tau}$; the mission failure probability, its predictive mean $\mathbb{E}[p_\tau(\lambda) \mid \mathbf{y}]$; the predictive survival, $R_{\text{pred}}(\tau \mid \mathbf{y}) = 1 - \bar{p}_\tau$ (Chapter 15).
p^*, λ^*, τ_p	–, yr^{-1}, yr	A limit on a failure probability, 5 %; the rate at which p_τ reaches it, $-\ln(1-p^*)/\tau$; the window length at which the unit's own failure probability reaches it, $-\ln(1-p^*)/\lambda$ (Chapter 15).
τ_{sd}	yr	The time from window 36 to the October shutdown, 0.75 yr: the planning date t_{sd} as a window length (Chapter 15).
n_ℓ, E_ℓ, μ_ℓ	–, coil-yr, –	Count and exposure of count row ℓ and its expected count $E_\ell\lambda$: Chapter 15's four rows on the age band, the fourth the prior.
Λ, $\sigma_{\omega,a}$, ΔE	–, –, coil-yr	The information on the log-rate, $a_\lambda + k$, the sum of the rows' expected counts; the log-rate's spread after a candidate log of ΔE further coil-years in the band (Chapter 15).
ω_m, ω_q, λ_q	–, –, yr^{-1}	A node of the grid on the log-rate, weighted by ϖ_m; the q-quantile of the log-rate and of the rate, its image by the quantile map (Chapter 15).
$\lambda_{\text{pred}}(\tau)$	yr^{-1}	The predictive hazard after a time τ without a failure, $a'_\lambda/(b'_\lambda + \tau)$ (Chapter 15).
M_{own}, M_{sh}, $B_{\text{own,sh}}$, $\mathbf{y}_0$, $\mathbf{y}_{\text{sib}}$	–	The drawings in which coil 0's year in the band has its own rate and in which it shares the siblings'; the Bayes factor between them; coil 0's own record and the siblings' (Chapter 15).
$\bar{a}_a$, μ_a, N_f, a_w	yr, –, –, yr	The mid-age of bin a, $a + \frac{1}{2}$; its expected count $E_a\lambda(\bar{a}_a)$; the log's total failures, 32; the failure-weighted mean age $\sum_a n_a\bar{a}_a/N_f$, the age of the band's waist (Chapter 16).
$\mathbf{X}_a$, $\mathbf{k}$, a_{o}	–, –, yr	The log-count Jacobian $\partial \ln \boldsymbol{\mu}_a/\partial \mathbf{z}$, with rows $(1, \bar{a}_a)$; the bins' counts $(n_0, \ldots, n_{11})$; an origin of age at which the level is read, $\ln\lambda(a_{\text{o}})$ (Chapter 16).
$\mathcal{E}(\theta_a)$, Γ_{reg}, θ_m	coil-yr, –, yr^{-1}	The age-weighted exposure $\sum_a E_a\mathrm{e}^{\theta_a\bar{a}_a}$, the rate of the level's gamma conditional $\text{Gamma}(N_f, \mathcal{E}(\theta_a))$; the regularized incomplete gamma function; a node of the grid on the slope, weighted by ϖ_m (Chapter 16).

Symbol	Unit	Meaning
t_{dbl}, t_{lim}, $I_\tau(\theta_a)$	yr, yr, yr	The doubling time of the rate, $\ln 2/\theta_a$, infinite for $\theta_a \le 0$; the span within which a doubling is asked (Chapter 16); the integral of $e^{\theta_a a}$ over ages 3 to $3+\tau$, so that coil 0's expected count over the span is $\lambda_0 I_\tau(\theta_a)$.
s_θ, M_{age}, M_{const}, B_{age}	yr^{-1}, –, –, –	The prior spread of the slope θ_a in the evidence; the drawings of the age curve and of the constant rate, $\theta_a = 0$; the Bayes factor of the first against the second (Chapter 16).
θ_g, n_{ia}, μ_{ia}, $g_i(a)$	–, –, –, –	The log-rate's coefficient on a coil's log-derate, a covariate; coil i's failures and expected count in its year of age a; its line $g_{i,0} + \gamma_i a$ from Chapter 9's fit (Chapter 16).
θ_{aa}, c_{p}, $\theta_{\ln a}$	yr^{-2}, –, –	The quadratic coefficient of a log-rate curved in age; the level and the exponent of the power law in age (Crow–AMSAA), $\ln\lambda = c_{\text{p}} + \theta_{\ln a}\ln a$: the curve-form checks of Chapter 16.
Z_i, ν, $\mu_{i\cdot}$	–	Coil i's gamma frailty, its multiplier on the rate, mean one; the frailty's coefficient of variation, the unit-to-unit spread of failure proneness; coil i's expected total over its years (Chapter 16).
E_b^r, $\mathbf{J}_r$, $\mathbf{\Sigma}_r$	coil-yr, varies, varies	The exposure a candidate record r adds in age bin b; its Jacobian lines; the preposterior covariance $(\mathbf{\Sigma}^{-1} + \mathbf{J}_r^{\mathsf{T}}\mathbf{J}_r)^{-1}$ (Chapter 16).
a_i, t_i^*, $g_i(a)$	yr, yr, –	Fleet coil i's age today; the time after today at which its log-derate reaches g_F, negative (-1 yr) for a coil already past it and ∞ for one whose line does not fall; its log-derate at age a, $g_{i,0} + \gamma_i a$ (Chapter 20).
$\mathbf{x}_i$, $\hat{\mathbf{x}}_i$, $\boldsymbol{\ell}_m$	–	Coil i's pair $(g_{i,0}, \gamma_i)$ and its conditional mean (Chapter 9); node m of the spreads' grid $(\ell_{\text{c}}, \ell_\gamma)$, of weight ϖ_m (Chapters 9 and 20).
$f_{1,i}$, $f_{2,i}(t_{\text{ah}})$, $\mathbf{g}_{1,i}$, $\mathbf{g}_{2,i}(t_{\text{ah}})$	–	Coil i's margin over the functional limit today and t_{ah} years ahead, $g_i(a_i) - g_F$ and $g_i(a_i + t_{\text{ah}}) - g_F$; their gradients on $\mathbf{x}_i$, $(1, a_i)$ and $(1, a_i + t_{\text{ah}})$ (Chapter 20).
$\bar{f}_{1,i}$, $\bar{f}_{2,i}(t_{\text{ah}})$, u_1, u_2, $u_{1,i,m}$	–, –, –, –, –	The margins' means at a node of the grid; their standardized levels $-\bar{f}/\sqrt{\text{Var}(f)}$; u_1 for coil i at node m (Chapter 20).
$P_i(t_{\text{ah}})$, G_F	–, kW/K	Probability that coil i crosses its functional limit after today and by time t_{ah}, $\Pr(0 < t_i^* \le t_{\text{ah}} \mid \mathbf{y})$; the functional limit on Part I's hall, 5.0 kW/K on its design value, 5.8333 kW/K on a 7 kW/K coil (Chapter 20).
The UPS battery string		
$C_{\text{bat},t}$, $R_{\text{bat},t}$	Ah, mΩ	Usable capacity and ohmic resistance of the string in test t; $\bar{C}_{\text{bat}}$, $\bar{R}_{\text{bat}}$ their ratings, 100 Ah and 40 mΩ.
ξ_t, h_C, ΔR	–, –, mΩ	Log-capacity $\ln(C_{\text{bat},t}/\bar{C}_{\text{bat}})$; capacity against commissioning; resistance rise since commissioning.
v_0, Δv, h_{ch}, N	V, V, V, –	A block's loaded curve at the usable-empty point, its rise to full, and the charge curve's offset above it; the number of blocks, 40.
I_d, I_c, SOC	A, A, –	Discharge and recharge currents; state of charge.

Symbol	Unit	Meaning
$V_{d,t}$, $\dot{V}_{d,t}$, $V_{c,t}$, $\dot{V}_{c,t}$	V, V/h	Level and slope of test t's discharge line and of its recharge line.
$T_{\mathrm{run},t}$, V_{cut}, $\mathrm{SOC}_{\mathrm{cut},t}$, $V_{\mathrm{h},t}$	h, V, –, V	Run time to the cutoff; the cutoff on the string, 420 V; the state of charge there; the loaded head $N(v_0 + \Delta v) - V_{\mathrm{cut}} - I_d R_{\mathrm{bat},t}/1000$.
σ_V, $\sigma_{\mathrm{run},t}$, $\mathbf{s}$, $\mathbf{n}$	V, h, mixed	The voltmeter's spread; the run-time row's spread; the two discharge lines as coordinates; the ridge (null direction) of the discharge rows.
Probability		
$\Pr(A)$, $\Pr(A \mid B)$	–	Probability of event A; conditional on event B.
$p(\cdot)$	varies	A probability density; $p(\mathbf{z} \mid \mathbf{y})$ is the posterior of the state, $p(\mathbf{y} \mid \mathbf{z})$ the likelihood, $p(\mathbf{z})$ the prior.
$\mathbb{E}$, Var, Cov	varies	Expectation, variance, covariance.
$\mathcal{N}(\mu, \sigma^2)$	–	Normal distribution with mean μ and variance σ^2; Φ its standard cumulative distribution function, φ its density.
Inference objects		
$\mathcal{L}(\mathbf{z})$	–	The objective, minus the log posterior up to a constant.
$\hat{\mathbf{z}}$	varies	The maximum a posteriori (MAP) state, the minimizer of $\mathcal{L}$.
$\mathbf{J}$	varies	Jacobian of the normalized rows with respect to the state, evaluated at $\hat{\mathbf{z}}$.
$\mathbf{\Lambda}$	varies	Information matrix (Gauss–Newton Hessian of $\mathcal{L}$), $\mathbf{J}^{\mathsf{T}}\mathbf{J}$ for normalized rows.
$\mathbf{\Sigma}$, Σ_{ij}	varies	Posterior covariance of the state under the Laplace approximation, $\mathbf{\Lambda}^{-1}$; its entries. The posterior spread of entry i is written $\sqrt{\Sigma_{ii}}$.
$f(\mathbf{z})$, $\mathbf{g}$	varies	A derived quantity of the state and its gradient $\partial f/\partial \mathbf{z}$ at $\hat{\mathbf{z}}$.
$\mathbf{N}$, $\mathbf{J}_{\mathrm{red}}$	varies	Null-space basis of the physics rows; the Jacobian of the reduced state (Chapter 4).
f_L, f_U	varies	The ends of the credible interval of a derived quantity.
ϱ, $\boldsymbol{\varrho}$, Φ_2	–	A correlation coefficient (Chapters 2 and 3; in Chapter 24 that of two consecutive logged readings' errors); the posterior's correlation matrix (Chapter 7); the distribution function of two standard normals with correlation ϱ.
υ, $\mathbf{\Upsilon}$, S_{in}	–	Weight of a reading; the gain of the correction form; the innovation variance of one reading (Chapter 4).
x_j, w_j	–	Gauss–Hermite nodes and weights.
ϖ_j, N_{eff}, p_{L}	–	Weight of draw j in the reweighted draws; their effective sample size $1/\sum_j \varpi_j^2$; the Laplace normal used as the proposal (Chapter 5).
ℓ_t, ε_q	–	Magnitude of the slope of window t's reading row; the summed departure of unread windows from the line (Chapter 4).
y_{new}, $\tilde{y}_a$	varies	A future reading (the predictive) and the outcome of a candidate measurement a before it is taken (the preposterior).

Symbol	Unit	Meaning
U_{p}, $U(a)$	varies	A quantity of the posterior after $\tilde{y}_a$; its expectation over the outcomes, which ranks the candidates (Chapter 2's design form).
M, M_j, B_{AB}	–	A drawing (model) of the plant; the members of a family of candidates; the Bayes factor $p(\mathbf{y} \mid M_A)/p(\mathbf{y} \mid M_B)$.
M_{clean}, M_{fouled}, M_{bias}, B_{fc}, B_{fb}	–	Chapter 2's drawings of a warm condition test: the air path clean, the air path fouled, the hall sensor with a bias; the Bayes factors of fouled against clean and of fouled against bias.
M_{one}, M_{two}, $B_{\mathrm{two,one}}$	–	Chapter 4's alternatives for the hall's air path, a single thermal resistance and two distinct heat-removal paths (beside the sensor bias M_{bias}); the Bayes factor between them.
$\mathbf{J}_a$, $\mathbf{\Lambda}_a$, $\mathbf{\Sigma}_a$	varies	The Jacobian rows of a candidate measurement, its information $\mathbf{J}_a^{\mathsf{T}}\mathbf{J}_a$, and the preposterior covariance $(\mathbf{\Sigma}^{-1} + \mathbf{\Lambda}_a)^{-1}$.
P_M, $\mathcal{M}$, $\mathcal{M}_k$	–	The posterior probability $\Pr(M \mid \mathbf{y})$ of a drawing, its weight in the mixture; a group of drawings that share one factor, the curvature state or the step's kind (Chapter 14); the groups of one open question, whose partial perfect information Chapter 17 values.
$\hat{\mathbf{z}}_M$, $\mathbf{\Sigma}_M$, $\mathbf{\Sigma}_M^{[\cdot]}$, $\mathbf{\Sigma}_{M,a}$	varies	The MAP state and Laplace covariance of drawing M; its block on the entries the forecast reads (five, $(g_{1,36}, g_{2,36}, \gamma_1, \gamma_2, \dot{\gamma}_1)$, in Chapter 14; seven with the commissioning entries in Chapter 17); its preposterior covariance after candidate a, $(\mathbf{\Sigma}_M^{-1} + \mathbf{J}_a^{\mathsf{T}}\mathbf{J}_a)^{-1}$ (Chapter 14 on).
$\mathbf{g}_{t^*}$, $\mathbf{g}_*$, $\mathbf{g}_s$, $\mathbf{g}_G$	varies	The pattern of the crossing's gradient on a drawing's state; its gradient on the five forecast entries at the MAP, behind the delta band; the gradient of $G_{\mathrm{tot},s}$ on those entries; that of $G_{\mathrm{tot},36}$ on today's log-derates (Chapter 14).
$\mathbf{y}_{0:t_{\mathrm{as}}}$, $\tilde{\mathbf{y}}_{37:45}$	varies	The readings of windows 0 to t_{as} with the prior rows, an as-of fit's evidence; the readings of windows 37 to 45 not yet logged, drawn from their predictive (Chapter 19).
$\boldsymbol{\theta}_t$, $\bar{\boldsymbol{\theta}}_t$, $\bar{\boldsymbol{\theta}}_t^-$	mixed	The carried entries $(g_{1,t}, g_{2,t}, \gamma_1, \gamma_2, \dot{\gamma}_1, \beta_1, \dot{b}_{d,1}, \delta)$, all a later window's rows touch; their mean after window t's rows; before them, with the trend applied (Chapter 19).
$\mathbf{\Sigma}_t$, $\mathbf{\Sigma}_t^-$, $\mathbf{T}_t$, $\mathbf{S}_t$	varies	Covariance of $\boldsymbol{\theta}_t$ after and before window t's rows; the trend rows' transition from window t to $t+1$; the innovation covariance $\mathbf{J}_t\mathbf{\Sigma}_t^-\mathbf{J}_t^{\mathsf{T}} + \mathbf{I}$ of a look, with the gain $\mathbf{\Upsilon}_t$ of the correction form (Chapter 19).
$P_M(t)$, $\mathbf{m}$, $\mathbf{V}$, $m_{1\mid2}$, $s_{1\mid2}$	–, –, –, –, –	A drawing's probability after the rows of window t, $P_M(36) = P_M$; mean and covariance of $(g_{1,t_{\mathrm{as}}+1}, g_{2,t_{\mathrm{as}}+1})$ one window ahead; the conditional mean and spread of g_1 given g_2 in the region's integral (Chapter 19).

Symbol	Unit	Meaning
$\mathbf{A}$, $\boldsymbol{\Sigma}_0$	varies	The map from the state to the quantities a block of correlated readings reads, and the prior covariance, in the precision $\boldsymbol{\Sigma}^{-1} = \boldsymbol{\Sigma}_0^{-1} + \mathbf{A}^{\mathsf{T}}(\sigma^2\mathbf{R})^{-1}\mathbf{A}$ (Chapter 24).
Decisions		
a, $L(a, \mathbf{z})$	–, R$	An action (repair, replace, inspect, wait); its loss when the state is $\mathbf{z}$.
$\mathcal{R}(a)$, c_a, C_e	R$	The expected loss of action a, $\mathbb{E}[L(a, \mathbf{z}) \mid \mathbf{y}]$; its fixed cost; the consequence of event e.
c, c^*, c_1^*	varies, –, –	A decision level on a monitored parameter (the work-order trigger), from Chapter 19 a level on the probability $q_{t_{\text{as}}}$; the level of least expected loss with a refit every month, 0.0197; the one-look level $C_{\text{false}}/(C_{\text{false}} + C_{\text{miss}})$, 0.0980 (Chapter 19).
κ	–	Assurance level of an acceptance rule: the posterior probability a requirement must reach (0.95 in Chapter 22's verdict; Chapter 21's break-even p_{be} replaces it with a level the costs set).
EVSI, EVPI, EVPPI	R$	Expected value of sample information: what a measurement is worth before an action. Expected value of perfect information, $\mathcal{R}(a^*) - \mathbb{E}[\min_a L(a, \mathbf{z}) \mid \mathbf{y}]$: what knowing the whole state would be worth, the most any measurement can be (Chapter 17, its general form; Chapter 20 specializes it to the fleet's orders). Expected value of partial perfect information: the same with one open question known instead of the whole state (Chapter 17).
SF, OW, FO, RF, FC	R$, R$, R$, –, R$	Part V's cost schedule, each a stated input: a month with the hall at or below G^* (SHORTFALL, 150 000 per month; Chapters 17–19); the premium for work away from an October outage (OFF_WINDOW, 12 000 a visit; Chapters 17–19); a work order raised while the hall would have held (FALSE_ORDER, 3 000; Chapters 18 and 19); the share of the loss since commissioning that a clean restores (RESTORE_FRACTION, 0.85; Chapter 17, named in words in Chapter 20); a fleet coil crossing its functional limit before its visit (FLEET_CROSS, 40 000; Chapter 20). CLEAN (8 000 a coil), REPLACE (60 000 a coil) and INSPECT (1 500 a crew visit at reference grade) are named in words.
$\mathcal{P}$, $\mathcal{P}^*$, $\mathcal{P}_{\text{v}}$, $\mathcal{P}_{\text{dash}}$, $\mathcal{P}_{\text{cal}}$, $\varnothing$	–	A maintenance plan, the work at each visit of the horizon (one of 256); the best plan of the chosen action a^*; a one-visit plan; the dashboard's and the calendar's schedules; the plan of no work (Chapter 17). An action a is a class of plans, priced by its best plan.
$\mathcal{O}$, t_{v}, $W_t(\mathcal{P})$, $c_{\mathcal{P}}$	window, window, R$, R$	The October outages of the horizon, $\{46, 58, 70, 82, 94\}$; a visit's window; the price of the work plan $\mathcal{P}$ does at window t (CLEAN for each coil cleaned, REPLACE for each replaced); the plan's discounted work with the off-window premium, its c_a (Chapter 17).

Symbol	Unit	Meaning
$\mathrm{df}(t)$, $\mathcal{U}_t$	–, –	The discount factor to today, $1.08^{-(t-36)/12}$; the shortfall event of month t, $\{G_{\mathrm{tot},t} \le G^*\}$, whose consequence is $\mathrm{SF}\,\mathrm{df}(t)$ (Chapter 17).
$d^+_{u,t}$, $g^{\mathcal{P}}_{u,t}(\mathbf{z};\mathcal{P})$, $G_{\mathrm{tot},t}(\mathbf{z};\mathcal{P})$	–, –, kW/K	Coil u's derate factor after a clean at window t, $d_{u,0} - (1-\mathrm{RF})(d_{u,0} - d_{u,t})$; the value of $g_{u,t+1}$ at which the row from window t is zero under plan $\mathcal{P}$; the coils' total conductance once the plan's work is done (Chapter 17).
S_{sd}, $\mathbf{g}_L$	R\$, varies	The expected discounted shortfall from window 37 to the shutdown with no work, set against OW; the pattern of the loss on the extended state, whose gradient is zero wherever it is defined (Chapter 17).
G_{P}, m_{P}, t_{P}	kW/K, kW/K, window	The potential-failure level $G^* + m_{\mathrm{P}}$, the P of the P–F interval, at which a condition test raises the work order, 11.261 kW/K; its margin over the capacity limit, 0.5 kW/K; the first passage of $G_{\mathrm{tot},t}$ to G_{P} (Chapter 18).
Δ_{PF}, $\mathbf{g}_{\mathrm{PF}}$	window, varies	The P–F interval of a path, $t^* - t_{\mathrm{P}} \ge 0$, in months; its gradient on the forecast entries for the delta band (Chapter 18).
Δ_{I}, s_1, $\mathcal{S}$	window, window, –	An inspection interval; the window of a schedule's first test, its phase; the set of test windows, $\mathcal{S}(\Delta_{\mathrm{I}}, s_1) = \{s_1, s_1 + \Delta_{\mathrm{I}}, \ldots\}$ or a dated set (Chapter 18; Chapter 22's $\mathcal{S}$ is likewise a schedule of looks).
$\hat{G}_s$, σ_{I}, $p_{\mathrm{no},s}$	kW/K, kW/K, –	A condition test's estimate of $G_{\mathrm{tot},s}$; its standard deviation, 0.1033 kW/K at the reference grade; the probability, given the path, that the test at window s raises no order, $\Phi((G_{\mathrm{tot},s} - G_{\mathrm{P}})/\sigma_{\mathrm{I}})$ (Chapter 18).
$\mathbf{r}_{\mathrm{I}}$, $\mathbf{z}_{\mathrm{I}}$, $\mathbf{J}_{\mathrm{I}}$, $\mathbf{\Sigma}_{\mathrm{I}}$, $\mathbf{g}_G$	varies	A condition test's rows on its window's four entries $(Q_s, g_{1,s}, g_{2,s}, \beta_1)$; those entries; the rows' Jacobian and covariance; the gradient of $G_1 + G_2$ on them, with $\sigma_{\mathrm{I}}^2 = \mathbf{g}_G^{\mathsf{T}} \mathbf{\Sigma}_{\mathrm{I}} \mathbf{g}_G$ (Chapter 18).
P_{U}, Δ_5, Δ^*	–, window, window	Probability of an undetected crossing: of an interval, averaged over its phases and the drawings, or of a dated set; the longest interval that keeps it, and every shorter one's, at or below 0.05; the interval of least expected cost (Chapter 18).
C_V, N_V, N_{sf}, $s^+(t^*)$	R\$, –, window, window	The price of one test visit reading both coils, two INSPECTs, 3 000; the number of visits of a program; its months at or below G^*; the first test window at or after the crossing (Chapter 18).
$\mathcal{U}$, $q_{\mathcal{U}}$	–	The event a work-order decision turns on, the hall below its limit at a window an order raised now would protect, the shortfall event $\mathcal{U}_t$ of that window; its posterior probability (Chapter 19).
$q_{t_{\mathrm{as}}}$, Δ_{L}	–, window	The trigger's probability at look t_{as}, $\Pr(G_{\mathrm{tot},t_{\mathrm{as}}+\Delta_{\mathrm{L}}} \le G^* \mid \mathbf{y}_{0:t_{\mathrm{as}}})$, $t_{\mathrm{as}} = 36, \ldots, 45$; the lead time from an order to the work, one window (Chapter 19).

Symbol	Unit	Meaning
C_{false}, C_{miss}	R$	The extra cost of an order when the event does not occur, FO + OW, and of waiting when it does, SF − OW (Chapter 19).
$T_{\text{ord}}(c)$, $\mathcal{R}(c)$, $\mathcal{R}_a(c)$	window, R$, R$	The first look at which $q_{t_{\text{as}}} \geq c$, window 46 if none; the rule's expected loss over the records not yet logged; the same with candidate measurement a's rows (Chapter 19).
$\text{OC}(c;\ \cdot)$	–	Operating characteristic of a rule: its probability of firing at one look or more over records drawn at a stated truth, $\text{OC}(c;$ no loss) for the false orders (Chapter 19; Chapter 22 writes $\text{OC}(z_i^{\text{t}}; a)$ for a monitoring plan).
$\mathcal{V}$, v_i, $t_{\text{ah},v}$	–, week, yr	A visiting order of the fifty fleet coils; the week in which it visits coil i; the time of week v after today, $(v-1)/52$ (Chapter 20).
$\mathcal{V}^*$, $\mathcal{V}'$, $\mathcal{V}_{\text{dash}}$, $\mathcal{V}_{\text{now}}$	–	The order of least expected consequence, by linear assignment; another order; the dashboard's worst-looking-first order; the present plan, before a candidate's outcome (Chapter 20).
X_{iv}, v_i^*	–, week	The assignment matrix, one when coil i is visited in week v; the week in which coil i crosses on a draw, $\lceil 52\, t_i^* \rceil$, so that a visit in week v comes too late when $1 \leq v_i^* \leq v-1$ (Chapter 20).
$\mathcal{W}_i(\Delta)$, Δ	R$, yr	Cost of waiting of coil i over a delay, FC $P_i(\Delta)$; the delay (Chapter 20).
γ_{S}	yr^{-1}	The maker's specification as a threshold on a coil's own rate, $\ln(1-0.03) = -0.030459$: faster than specification when $\gamma_u < \gamma_{\text{S}}$ (Chapter 21).
P_u, P_u^0, B_u^{S}	–	Posterior and prior probabilities that coil u is faster than specification; the Bayes factor of the specification, posterior odds of the two halves over their prior odds (Chapter 21).
M_{fast}, M_{within}	–	The claim's drawing with γ_u's prior cut to $\gamma_u < \gamma_{\text{S}}$, and to $\gamma_u \geq \gamma_{\text{S}}$, each renormalized (Chapter 21).
CF, CV, p_{be}	R$, R$, –	Cost of filing a claim, R$ 5 000; value of a granted claim, a replacement coil, R$ 60 000; the break-even probability CF/CV = 1/12 above which filing has the lower expected loss, in place of an assurance level κ (Chapter 21).
$M_{2,15}^{(d)}$, q_d	–	The March step drawing with the curvatures of the coils in $d \in \{0, 1, 2, 12\}$ freed (none, coil 1, coil 2, both); its probability within the reported family of four (Chapter 21).
M_{arg}, M_{claim}	–	A counter-argument's drawing (the maker's alternative account of the record); the claim's drawing (Chapter 21).
$\hat{\gamma}_{u,d}$, $\varsigma_{u,d}$	yr^{-1}	Mean and spread of γ_u under drawing $M_{2,15}^{(d)}$ (Chapter 21).

Symbol	Unit	Meaning
γ_u^{td}, γ_u^{5y}	yr^{-1}	The loss to date, $(g_{u,36} - g_{u,0} - \mathbf{1}[u = 2]\,\delta)/(36\,\Delta t)$, the conceded step removed; the five-year rate, the mean of $\gamma_u(t_{\mathrm{y}})$ over the specification's five years, $\gamma_u + 1.0\,\mathrm{yr} \cdot \dot{\gamma}_u$ (Chapter 21).
$\mathbf{g}_{\mathrm{td},u}$, $\mathbf{g}_{\mathrm{5y},u}$	yr^{-1}	The gradients of the loss to date and of the five-year rate on the state (Chapter 21).
EVPI_u, EVSI_u	R$	A claim's expected value of perfect information, $\mathrm{CF}(1 - P_u)$, the cost of a claim that fails; a candidate measurement's expected value of sample information on coil u's claim (Chapter 21).
ς'_u, $\hat{\gamma}'_u$, P'_u	yr^{-1}, yr^{-1}, –	The spread of γ_u after a candidate measurement; the posterior mean it will leave, $\mathcal{N}(\hat{\gamma}_u, \varsigma_u^2 - \varsigma_u'^2)$ before its outcome; the probability of a breach after it (Chapter 21).
$\mathcal{D}(\Delta_{\mathrm{d}})$, $\mathcal{R}_{\mathrm{proof}}$	–, R$	The discount factor of a payment Δ_{d} years ahead at 8 % a year, Δ_{d} the delay; the expected loss of waiting under a standard of proof, by which a claim is filed only once $P'_u \geq 0.95$ (Chapter 21).
The value of monitoring		
ς_u, $\varsigma_u(t_{\mathrm{as}})$	yr^{-1}	The posterior standard deviation of coil u's rate, the spread of its rate (Chapters 21 and 22); in Chapter 22 the predictive spread of a record to window t_{as}, $\sqrt{[\boldsymbol{\Sigma}_{t_{\mathrm{as}}}]_{\gamma_u\gamma_u}}$, fixed before the record is read and equal to the spread of the rate's estimate over the records the plan may log. Chapters 7 and 11's ς_j is a quantity's standard deviation under drawing M_j, the same object within one drawing.
γ_{D}, $\gamma_{\mathrm{MD},u}(t_{\mathrm{as}})$	yr^{-1}	The stated loss a plan must detect, $\ln 0.95 = -0.05129$; the minimum detectable rate, $(z_\kappa + z_{\mathrm{POD}^*})\,\varsigma_u(t_{\mathrm{as}})$ (Chapter 22).
$\mathrm{OC}(z_i^{\mathrm{t}}; a)$, OC^*, z_i^{t}	–	Operating characteristic of a measurement or plan a: the probability that its outcome meets an acceptance rule when entry i of the state has the stated value z_i^{t}; the level it must reach (Chapter 22; Chapter 19's $\mathrm{OC}(c;\ \cdot)$ is the same object for a trigger).
$\mathrm{POD}_u(t_{\mathrm{as}})$, $\mathrm{PFA}_u(t_{\mathrm{as}})$, POD^*	–	Probability of detection of coil u's loss by the record to window t_{as}, the operating characteristic at $\gamma_u = \gamma_{\mathrm{D}}$; the false-alarm probability, the same at $\gamma_u = 0$; the power asked, 0.9 (Chapter 22).
z_κ, z_{POD^*}	–	The standard normal quantiles $\Phi^{-1}(\kappa) = 1.6449$ and $\Phi^{-1}(\mathrm{POD}^*)$ (Chapter 22).
$T_{90,u}$, $T_{90,u}^B$	window	Record length for a power of 0.9: the first window at which POD_u reaches POD^*; the same under the Bayes-factor rule (Chapter 22).
$\boldsymbol{\Sigma}_{t_{\mathrm{as}}}$, $\mathbf{z}_{t_{\mathrm{as}}}^{\mathrm{D}}$, $\hat{\gamma}_{u,t_{\mathrm{as}}}$	varies, varies, yr^{-1}	Covariance of a record to window t_{as} not yet logged; the hypothetical state at which it is linearized, the coils on the stated loss's line; the rate's estimate from that record (Chapter 22).

Symbol	Unit	Meaning
$\mathbf{J}_{t_{\text{as}}}$, $\mathbf{J}_{\text{pr}}$, $\mathbf{J}_{0:t_{\text{as}}}$, $\mathbf{J}_{r,t_{\text{as}}}$, $\boldsymbol{\varepsilon}_{t_{\text{as}}}$	varies	Jacobian of the record to window t_{as}; its four prior rows' lines; its reading and trend rows' lines; its reading rows' lines alone; the normalized errors of its rows, $\text{Cov}(\boldsymbol{\varepsilon}_{t_{\text{as}}}) = \mathbf{I}$ (Chapter 22).
$\mathcal{S}$, $\text{P}_{\mathcal{S},u}$	window, –	In Chapter 22 the schedule of looks, the windows at which the growing record is refitted (Chapter 18's set of test windows); the probability that the verdict passes at one look of $\mathcal{S}$ or more, the false alarm of repeated looks at $\gamma_u = 0$ (Chapter 22).
σ_γ, B^*, ς_{tr}, $\hat{\gamma}_{\text{tr}}$	yr^{-1}, –, yr^{-1}, yr^{-1}	The trend's prior spread in the Bayes-factor rule, 0.10; the stated level of the factor, 19; the spread and mean of the rate under that prior (Chapter 22).
$z_u^B(t_{\text{as}})$, POD_u^B, PFA_u^B	–	The Bayes-factor rule as a threshold on $-\hat{\gamma}_{u,t_{\text{as}}}/\varsigma_u(t_{\text{as}})$, growing with the record; its probability of detection and false alarm (Chapter 22).
$\varsigma_u^r(t_{\text{as}})$, POD_u^r, PFA_u^r	yr^{-1}, –, –	The estimate's spread from the reading rows' errors alone, for a truth with no wander; the probability of detection and false alarm it gives (Chapter 22).
$\boldsymbol{\Sigma}_{a,t_{\text{as}}}$, $\varsigma_u'(t_{\text{as}})$	varies, yr^{-1}	Preposterior covariance of the record to window t_{as} with candidate instrument a; the predictive spread it leaves (Chapter 22; Chapter 21's ς_u' at window 36).
$\mathcal{B}_a$, $\mathcal{B}_{M,a}$, $\bar{\mathcal{B}}_a$, $\mathcal{B}_{\text{cont}}$	window	The 5–95 % band of the crossing window t^* from the refit at window 45 under candidate a, mixed over the family at today's P_M; drawing M's own band; the own bands weighted, $\sum_M P_M \mathcal{B}_{M,a}$; the band under the record's continuation alone (Chapter 23).
$\Delta\mathcal{B}_a$, $\Delta\bar{\mathcal{B}}_a$	window	The value of candidate a for the date: the windows it takes off the family's band and off the own bands beyond the continuation (Chapter 23).
$F_{t^*}(s)$, $F_{t^*,a}(s)$	–	The distribution function of the crossing window, $\Pr(t^* \le s \mid \mathbf{y})$ (Chapter 14); the family's under candidate a, $\sum_M P_M \Pr(t^* \le s \mid \mathbf{z}_M \sim \mathcal{N}(\hat{\mathbf{z}}_M, \boldsymbol{\Sigma}_{M,a}))$ (Chapter 23).
$\mathbf{J}_{\text{now}}$, $\mathbf{J}_{\text{cont}}$	varies	The lines of the record's rows to window 36 and of every prior and trend row to window 45; those of the existing readings of windows 37 to 45, the continuation, from which each drawing's preposterior $\boldsymbol{\Sigma}_{M,a}$ at the refit is built (Chapter 23).
$\mathbf{z}_{M,45}$	varies	Drawing M's state carried to the refit at window 45, with the water-flow factors ι_1, ι_2 appended (Chapter 23).
ι_u, C_w	–, kW/K	Log of coil u's chilled-water flow against the balancing report's, prior $\mathcal{N}(0, 0.10^2)$, constant from window 37; that report's chilled-water capacity rate per coil, 14.0 (Chapter 23).
$r_{W,u,t}$, $r_{d',t}$, $r_{\Delta,u,t}$, $r_{r',t}$, $r_{c,t}$, $r_{C,u,t}$	–	The candidates' rows in windows 37 to 45: a water-side heat meter on coil u; a second discharge sensor on unit 1; a water-side temperature difference on coil u, $D_{u,t}/(C_w \text{e}^{\iota_u})$; a new return-air sensor; a supply-air sensor; an airflow station on unit u (Chapter 23).

Symbol	Unit	Meaning
$\mathbf{\Sigma}_5$	varies	The covariance block of the five forecast entries $(g_{1,45}, g_{2,45}, \gamma_1, \gamma_2, \dot{\gamma}_1)$ at a drawing's MAP state, on which the crossing's gradient $\mathbf{g}_*$ acts (Chapter 23).
$\mathbf{g}_G$, $\mathbf{g}_{\dot{G}}$, $\mathbf{g}_{\Delta g}$, $\mathbf{g}_{\Delta\gamma}$, $\mathbf{g}_{\dot{\gamma}}$	varies	The gradients on the same five entries of the total's level, its rate of change, the split of the log-derates, the split of the rates and the curvature: the five directions of the date's variance (Chapter 23).
$\dot{G}_{\mathrm{tot},t}$	kW/K/yr	The rate of change of the coils' total conductance at window t (Chapter 23).
$\mathbf{Q}$	–	Rows of a set of directions $\mathbf{Qz}$ of the state taken as known exactly, in $\mathrm{Var}(f \mid \mathbf{Qz})$ (Chapter 23).
$c_a^{\times}$, $a^{\dagger}$	R$, –	The price at which candidate a would match the best permanent sensor $a^{\dagger}$ per real on the own bands, $\Delta\bar{\mathcal{B}}_a\, c_{a^{\dagger}}/\Delta\bar{\mathcal{B}}_{a^{\dagger}}$; c_a is the candidate's price, the fixed cost of the action of buying it (Chapter 23).
p_{46}	–	A predicted record's refit probability of a crossing by the shutdown, $\Pr(t^* \le 46 \mid \mathbf{y})$; the wrong-call chance at the outage is $\mathbb{E}[\min(p_{46}, 1-p_{46})]$ (Chapter 23).
n_{L}, $\Delta\vartheta$, ϑ_w	–, h, h	The logging rate, readings of each channel per window (1 monthly, 30 daily, 720 hourly); the logging interval $\vartheta_w/n_{\mathrm{L}}$; a window's span, 720 h (Chapter 24).
ϱ_1, ϱ	–	Correlation of two reading errors an hour apart, 0.9, the correlation time $-1/\ln\varrho_1 = 9.49$ h; the correlation of two consecutive logged readings' errors, $\varrho_1^{\Delta\vartheta/1\,\mathrm{h}}$ (Chapter 24).
e_i, ϵ_i	as the reading	The AR(1) error of reading i of a channel, $e_i = \varrho e_{i-1} + \epsilon_i$; its new part, independent of every earlier error, variance $\sigma_k^2(1-\varrho^2)$ (Chapter 24).
$y_{k,t,i}$, $\mathbf{y}_{k,t}$, $\bar{y}_{k,t}$, f_k	as the reading	Reading i of channel k in window t; the window's n_{L} readings stacked; their generalized least-squares mean, weights 1 at the ends and $1-\varrho$ inside; the model value the channel reads (Chapter 24).
$r_{k,t,i}$, $\mathbf{r}_{k,t}$, $\tilde{r}_{k,t,i}$, $\mathbf{R}$, $R_{ii'}$	–	The row of reading i; the window's n_{L} rows stacked; the whitened row, the new part of its error over its own standard deviation; the readings' correlation matrix, $\mathrm{Cov}(\mathbf{r}_{k,t}) = \mathbf{R}$, with entries $\varrho^{\lvert i-i'\rvert}$ (Chapter 24).
$\mathbf{j}_{k,t}$	varies	The one Jacobian line the n_{L} rows of a channel's window share, $\sigma_k^{-1}\,\partial f_k/\partial\mathbf{z}$ (Chapter 24).
$n_{\mathrm{eff}}(n_{\mathrm{L}})$, $n_{\mathrm{eff}}^{\mathrm{mean}}$	–	The effective number of readings of a window, $\mathbf{1}^{\mathsf{T}}\mathbf{R}^{-1}\mathbf{1} = (n_{\mathrm{L}} - (n_{\mathrm{L}}-2)\varrho)/(1+\varrho)$, which saturates at $1 + \frac{1}{2}\vartheta_w \ln(1/\varrho_1)$; the same for the simple average (Chapter 24).
f_{sh}, n^{AR}, $\tilde{n}$	–	The part of a reading's variance shared by the whole window; the AR(1) effective number of the remaining part; $n^{\mathrm{AR}}/(1-f_{\mathrm{sh}})$, so that $n_{\mathrm{eff}} = \tilde{n}/(1+f_{\mathrm{sh}}\tilde{n}) < 1/f_{\mathrm{sh}}$ (Chapter 24).

Symbol	Unit	Meaning
$\mathbf{J}_y$, $\mathbf{J}_o$, $\mathbf{J}_{y,\mathrm{f}}$	varies	The record's 185 reading rows; its other 76 rows (priors and trend rows, from window 37 every trend row); the reading rows of the forecast windows 37 to 45 (Chapter 24).
$\mathbf{\Sigma}_{n_\mathrm{L}}$, $\mathbf{\Sigma}^{\mathrm{line}}_{n_\mathrm{L}}$, $U(n_\mathrm{L})$	varies, varies, yr^{-1}	The preposterior of the record logged at rate n_L, $(\mathbf{\Sigma}^{-1} + (n_{\mathrm{eff}}(n_\mathrm{L}) - 1)\mathbf{J}_y^\mathsf{T}\mathbf{J}_y)^{-1}$; the same in the straight-line drawing, the readings' part; the rate's spread $\sqrt{[\mathbf{\Sigma}_{n_\mathrm{L}}]_{\gamma_u\gamma_u}}$, Chapter 2's utility $U(a)$ at that rate (Chapter 24).
n_p, $\mathbf{\Lambda}_Q$, $\mathbf{\Sigma}_\mathrm{h}$	–, kW^{-2}, varies	The weight of the record's past reading rows in year 4's preposterior, 1 as it stands and n_{eff} with the logger's archive read; the future loads' prior precision, $(1000\,\mathrm{kW})^{-2}$ on each; year 4's preposterior at hourly logging, the base of step 7 (Chapter 24).

Vocabulary

Plant graph The physical system as nodes (places where a conserved quantity balances), edges (paths along which it flows) and sources, with the relations that must hold among them.

State vector The collection of free quantities the fit estimates at once.

Pinned, free, known input A quantity treated as known (a boundary condition, or an input such as the supply temperature read by a precise instrument) against one carried in the state.

Residual, row A relation written so that it equals zero when satisfied: physics rows constrain the state, reading rows compare it with the records, trend rows tie one window's parameter to the next, prior rows carry handbook or commissioning knowledge.

Window The span of records fitted as one steady condition: one representative hour of a month here.

Health index A physical parameter of the model read against its commissioned value; never a score built from raw performance.

Condition monitoring Following the health index through the windows of a record.

Degradation rate The trend of the health index per year.

Remaining useful life The time until the health index crosses the level at which the asset can no longer do its job, as a date with a band.

P–F interval The time from the first detectable change of the health index to its crossing of the functional limit.

Condition-based maintenance Maintenance triggered by the health index crossing a decision level, rather than by the calendar.

Partial pooling Estimating each unit of a fleet with a shared prior whose mean and spread are themselves estimated, so that the fleet informs the unit and the unit informs the fleet.

Change-point detection Locating a step in a parameter's path by the evidence of a step freed at each candidate window.

State of health The capacity and internal resistance of a battery against their values when new.

Laplace approximation Replacing the posterior by the Gaussian whose covariance is the inverse curvature of the objective at the MAP state.

Delta method Linearizing a derived quantity around the MAP state so that its variance is $\mathbf{g}^\top \boldsymbol{\Sigma} \mathbf{g}$.

Posterior predictive The distribution of a future reading obtained by averaging its model over the posterior.

Evidence, Bayes factor The normalizing constant of one model's posterior; the ratio of two models' evidences.

Preposterior The posterior that would result from a measurement not yet taken; the object that values a measurement before it is made.

The five computations Posterior, trend, evidence, preposterior and expected consequence: the objects every asset-management question is answered from.

Part IV

Forecasting the Need for Attention

Chapter 14

Remaining Useful Life and Survival to the Next Outage

14.1 The dashboard's crossing date and the coils' remaining life

It is December of year 3, window 36, three years after the two-unit hall was handed over. The October shutdown of year 4, window 46, is the next outage at which the coils can be taken out of service for cleaning, and the planner must decide now whether the work goes into that outage or whether the hall cannot wait for it. The capacity study states the limit as a conductance: the two coils together must keep $G_1 + G_2 \geq 10.761\,\mathrm{kW/K}$ for rack 3 to stay at 27°C on the design day. The condition-monitoring dashboard gives each coil's health-index tile and its straight line, Chapter 10's: $1.0118 - 0.1576$ a year for coil 1 and $1.0154 - 0.1482$ a year for coil 2, with today's tiles at 0.4873 and 0.5947. The planner turns the lines into kW/K with the commissioning test's conductances, 6.7164 and 6.4638 kW/K, and gets $13.3588 - 2.0164\,t_\mathrm{y}\,\mathrm{kW/K}$ with t_y in years. That line reached the limit at $t_\mathrm{y} = 1.2884\,\mathrm{yr}$, window 15.46, in April of year 2, and today it reads 7.3097 kW/K, 3.4513 kW/K below the limit. The conclusion the chart invites is that the hall has been past its design capacity since April of year 2, and that both coils must be cleaned now, not at the October shutdown. With the rated 7 kW/K per coil as the reference the line crosses in August of year 2, window 19.22, and the conclusion is the same.

The chart cannot say whether the tiles' fall is the coils at all: Chapter 10 showed that a tile is a ratio of approaches, which falls with the load, the season and unit 1's drifting sensor. Nor can it say at what date the coils' own conductance reaches the limit, when its rate is uncertain and may itself be changing (Chapter 11), and when part of coil 2's loss may be a single step that does not continue (Chapter 12). And it cannot say how likely the hall is to hold until October, which is a probability, not a date. The question of this chapter is

when the sum of the two coils' conductances first falls below the capacity study's limit, read as the first passage of the path that the condition record's fit carries beyond the record, averaged over the drawings that Chapters 11 and 12 left open, with the probability that it holds until the October shutdown.

It is answered with the method of Chapter 5, on Chapter 10's stacked state extended to the forecast windows by the trend rows of Chapter 4, over the family of Chapters 11 and 12 averaged as in Chapter 7.

Jargon: forecast window, survival to an outage, forecast averaged over drawings, within- and between-drawing width

A *forecast window* is a window of the state past the last reading: it carries the parameters of that window and the trend rows that tie it to its neighbors, and no reading. *Survival to an outage* is the probability that the crossing comes after a stated planning date. A *forecast averaged over drawings* mixes the crossing's distribution under each drawing with the drawing's posterior probability as its weight. Its *within-drawing width* is the band each drawing gives on its own, averaged over the weights, and the *between-drawing width* is what the mixture's band adds to it. Chapter 2 defined the first passage, the plug-in date, the delta band, the band of a date by inversion and the mass that never crosses (Section 2.3.4); the box does not repeat them.

Key idea

A remaining life is a date read from the posterior of the path, not a line drawn through a chart. The physics puts the path on the coils' conductance, the trend rows carry it past the record with the wander they already allow, and the date is the first window at which the path falls below the limit: a quantity of the extended state, with a median and an inversion band but no mean. When the record has not settled how the path bends, the date is averaged over the drawings, and its band carries that indecision.

14.1.1 A date from the posterior of a path

Chapter 2 wrote the crossing of a level as the second line of the trend form (2.21), on a straight line with no wander, and worked it on Part I's hall (Figure 2.2). For a quantity that falls to its level, as a conductance does, the crossing by a date s there is the joint event that the trend is negative and the quantity stands at or below the level at s. That event is the first passage only because a straight line, once below, stays below. The path of this chapter is the trend rows' path, which wanders from month to month about its trend, so a path can dip below the level and come back. The crossing is then defined on the path itself. With $G_{\mathrm{tot},t} = G_{1,t} + G_{2,t}$ the coils' total conductance in window t, Chapter 13's equation (13.1), and G^* the limit, the first window at or below the limit is t_c, and the crossing time t^* is read between that window and the one before it by interpolating G_{tot} linearly:

$$t_c(\mathbf{z}) = \min\{t > 36 : G_{\mathrm{tot},t}(\mathbf{z}) \le G^*\}, \qquad t^* = t_c - 1 + \frac{G_{\mathrm{tot},t_c-1} - G^*}{G_{\mathrm{tot},t_c-1} - G_{\mathrm{tot},t_c}} \in (t_c - 1,\ t_c],$$

$$\{t^* \le s\} = \bigcup_{t=37}^{s} \{G_{\mathrm{tot},t} \le G^*\} \supseteq \{G_{\mathrm{tot},s} \le G^*\} \quad \text{for a window } s. \tag{14.1}$$

The date is the window $t_c = \lceil t^* \rceil$, with its month and year. The union counts a path that has been below the limit at any window up to s, whether or not it is below at s; the threshold at s alone counts only the paths below at s. The remaining useful life is t^* less today's window, $t^* - 36$, in months.

The windows after the record are entries of the state like the windows of the record. Each forecast window carries its two log-derates, tied to the previous window by the same trend row as the record's windows, and it has no reading. The record's entries keep their posterior, and each forecast window adds one month's wander to the trend the drawing carries. The crossing is a function of that extended state, and its distribution is read by the second line of equation (2.21) with the first passage in place of the joint event.

Principle 14.1. *A date is a ratio of a margin to a rate, so it has a median and a band but no mean, and its band is read by inverting the probability of having crossed by each date. A line drawn through raw performance gives a date with no band, on a quantity that is not the coils.*

14.1.2 The two-unit hall carried past its record (step 0)

The graph is Chapter 10's, Figure 10.1: Chapter 5's window graph at each of the 37 windows of the record, with its five physical rows (5.2) at that window's known inputs $T_s(t)$ and $Q_o(t)$; the 72 trend rows between consecutive windows; the two trend entries γ_1 and γ_2; the shared recirculation fraction β_1; and the drift node $\dot{b}_{d,1}$ on unit 1's discharge reading from window 20, equation (10.2). Chapter 11's curvature node $\dot{\gamma}_1$ hangs on coil 1's trend rows in the drawings that free it, and Chapter 12's step node hangs at window 15 in the drawings that carry a step, on the part the drawing names. The modeling choices of step 0 are Chapter 5's paragraph, carried through Chapter 10: steady state within each window, the airflow pinned with no discharge control, the season's quantities as known inputs, and one conductance per coil.

What the forecast adds to the graph. The forecast windows $37, 38, \ldots$ are path nodes $g_{1,t}$, $g_{2,t}$, with trend squares between them and no reading squares. Their IT loads do not enter: no reading row needs them, and the limit is stated at the design load. On every forecast window hangs a requirement node, $G_{\text{tot},t} \geq G^*$, with G^* a stated input, pinned. The shutdown is marked at window 46. One choice of step 0 is new here, and no question block owns it. *The forecast windows carry the record's trend rows unchanged*: the drawing's trend, its rate and, where it is freed, coil 1's curvature, with the wander $\sigma_q = 0.005$ per window. This buys a forecast that is the same fit, carried forward, with no second tool. It fails if the coils' condition changes by a cause the record has not seen, such as a cleaning, a valve restored or a new fouling regime; the forecast assumes no maintenance before the crossing.

The graph. The trend layer of both coils from window 34 on is the content of Figure 14.1. The upper chain is coil 1's path, the lower chain coil 2's; the black squares between neighbors are the trend rows. Windows 34 to 36 are read: each has a block between the chains for its load and five readings, drawn faintly because Figure 10.1 shows them. From window 37 on the windows have no block, and between the chains each carries its requirement node, linked to the pinned limit. Runs of windows are dashed boxes. The trend entries γ_1 and $\dot{\gamma}_1$ feed coil 1's squares from above and γ_2 feeds coil 2's from below, across window 36 as before it.

The record. The chapter reads the whole record as Chapters 10 to 13 read it, Table 10.1: window 0, the commissioning test, and windows 1 to 36, January of year 1 to December of year 3, five readings each. Three stated inputs enter here. The *limit* is the capacity study's $G^* = 10.761\,\text{kW/K}$ on $G_1 + G_2$: rack 3 at 27°C at 150 kW of IT load with the 15.5°C design summer supply. It is 0.8164 of the commissioning test's sum and 0.7686 of the rated 14 kW/K. The season enters the forecast through the limit's design supply, not through the trend, which Chapter 13 showed carries no season. The *planning date* is the October shutdown of year 4, window 46 (Oct Y4), ten windows ahead. And the *commissioning test's conductances*, its readings taken as exact (Chapter 5's $G_{u,\text{test}}$), 6.7164 and 6.4638 kW/K, sum 13.1803 kW/K, are what the dashboard uses to turn its tiles into kW/K.

The maintenance log so far. The two-unit hall's log holds the commissioning test of window 0; Chapter 5's condition assessment and Chapter 6's effort entry at window 18 (Jun Y2); Chapter 7's assessment at window 25 (Jan Y3), which held coil 1's cleaning order and

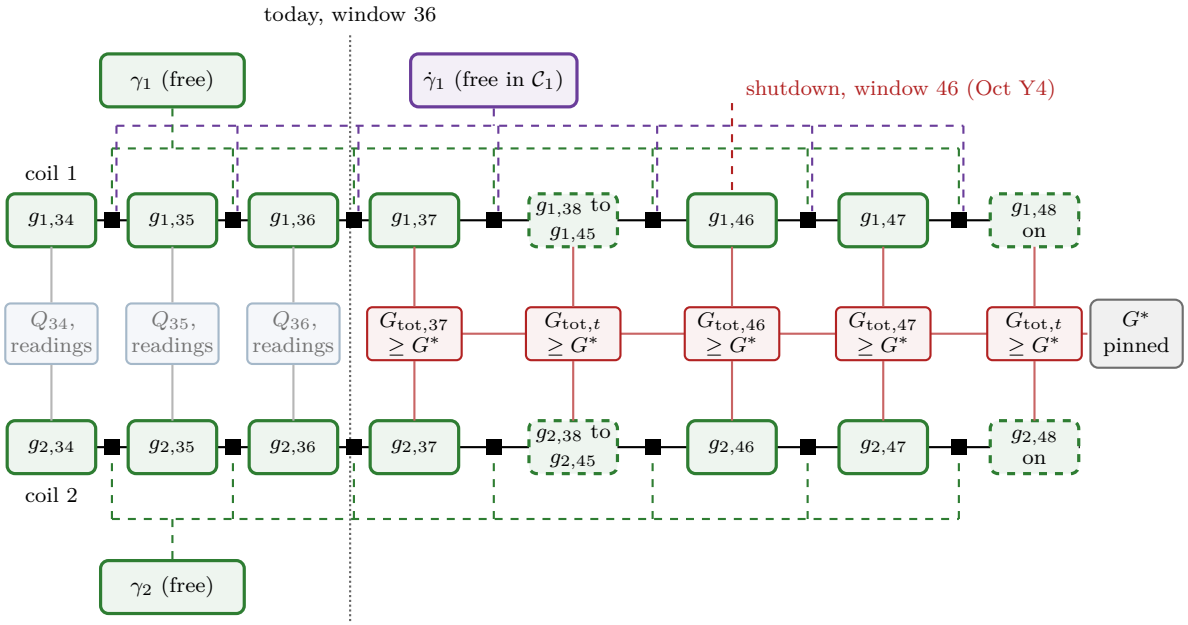


Figure 14.1: The trend layer of the two coils carried past the record, from window 34. Free (green): the log-derates $g_{u,t}$ of each coil, read windows 34–36 and forecast windows from 37 on (runs as dashed boxes); the trend entries γ_1, γ_2; and (purple) coil 1's curvature $\dot{\gamma}_1$ in the drawings that free it. Black squares: the trend rows (11.4), the same on either side of window 36. Faint blocks: the read windows' loads, physical rows and five readings (Figure 10.1); the forecast windows have none. Red: the requirement node $G_{\mathrm{tot},t} \geq G^*$ on every forecast window, linked to the pinned limit $G^* = 10.761\,\mathrm{kW/K}$, a stated input of the capacity study; the shutdown at window 46. Known inputs and pinned quantities of the read windows are not drawn; the step node of Chapter 12, at window 15, lies outside the windows drawn.

END OF THE PREVIEW

Asset Management by Derivation

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